## On Local Correlation and Partial Correlation for Measuring Strength of Local Linearity

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## Abstract

The classical Pearson correlation measures the linear association between two variables. A locally-weighted sample Pearson correlation is widely used in practice but little is known for its theoretical properties. In this paper, we investigate the locally weighted Pearson correlation using a kernel smoothing approach. We show that for bivariate data, the kernel-weighted correlation measures locally linear association and has an interpretation of the local standardized slope with fitting local linear regression. For trivariate data with kernel weights assigned by the third variable, the weighted correlation is a nonparametric analog of the local partial correlation, has connections to varying coefficient models, and measures the strength of local linearity controlling for the third variable. In both cases, their finite-sample and asymptotic properties are derived and the optimal orders of the bandwidth are provided. Simulated examples confirm the asymptotic theory and two data examples are given for illustration.

Keywords: kernel smoothing; local linear regression; varying coefficient model.